The Mathematical Myth

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1 The Rationals

The first mathematical attempt to heal the psychic split symbolised by the integers came in the early second millennium BCE, with the Egyptian invention of fractions. Why *fractions*? Well, fractions are, of course, extremely useful; their applications need no elucidation. Nonetheless, it is highly unlikely that a search for utility drove their inventors. Virtually all of the greatest mathematics became applicable only after the event, having started life as poetic creativity, as the whimsical struggle of some or other genius. Mathematics starts as art. And, if the history of art is anything to go by, some Bronze Age Egyptian number-priest invented fractions because doing so *felt good*. Playing with the ancient, divine symbolism of 1 and 2 gave that person a sense of God-connection, of $\Delta O_T > 0$, of transcendence, of *ātmatuṣṭi*.

Thus, in the Egyptian 12th dynasty, 1990–1800 BCE, the fractions came to fill the space between integer and integer, thereby symbolising a bridge. Until that point, 1 and 2 had been tally marks, separate by nature. Their very conceptual basis was their apartness: night and day, dark and light. But, with infinite shades of grey between, there was a soulish link, and symbolic progress had been made. With mathematical hindsight, this sounds occult, of course, but that is because of what number has *become*. Now, it is seen as pure rationality, but, in the late Bronze Age, it was esoterica. 1 and 2 were still pure magic, and mathematics was a form of witchcraft (as it yet remains to many): a way of approaching the old gods, a meditative, creative, *religious* discipline.
For the sensible, it remains so.

Why do mathematicians tend to be atheists?
Because they *already* talk to God.

By the time of Thales of Miletus (b. 624/623 BCE), there were infinitely many semi-divine fractions residing in the space between 1 and 2. The set of natural numbers had grown to become the set of *rational* numbers. But the

Ancient Greeks, ever exploring, came upon a problem. Following Pythagoras, the diagonal of a unit square was shown to be $\sqrt{2}$, which corresponded to no known ratio. When Hippasus suggested that it was, in fact, impossible to represent $\sqrt{2}$ thus—that $\sqrt{2}$ was *"irrational"*—it is said he was drowned at sea for his heresy. Apocryphal, probably, but the fact that the story exists at all is indicative: with the classic dogmatism of "the answer", the Pythagoreans, mathematical zealots, preached that all numbers were necessarily known.

2 The Reals

Some three hundred years later, the great Euclid proved decisively that $\sqrt{2}$ is not a rational number. As is always the way, what reason had previously known as impossible was made possible by new reason. There could be no argument. The supposed perfection of the rationals had been shown to be a false dawn: there were demons in the gaps, the irrationals to which Hippasus had once alluded. Of course, the old priests will have complained— *"Irrationals aren't real!"*—but time marched on regardless. And, with the Chinese development of negatives ("absurd", according to the Greek Diophantus) and the Indian invention of zero (atheism!), the number line was filled out fully. By the Middle Ages, number had become a continuum, and the symbolic chasm between 1 and 2 was spanned. And, this time, the healing was not just *hopefully* perfect, but it was *provably* perfect. All numbers were *demonstrably* among the known numbers, thus symbolic healing had gone as far as it could go. "This world is all there is," they said.

And, this time, they *knew* they were right.

But, while all of this ruckus was ensuing, the 9th century Persian mathematician *al-Khwarizmi* (who gave his name to the algorithm), working out of Golden Age Baghdad's wonderfully named House of Wisdom, invented algebra. *Al-jabr*, literally "completion", is the mathematical use of metasymbols, that is to say, the representation of unknown numbers (which are symbols in themselves, of course) by alphabetic characters. This is the deeper symbolisation that now sits at the heart of mathematics. With these higher-level "symbols of symbols", algebraic initiates wield a weapon of extraordinary power.

Al-Khwarizmi had created a monster.

The newly invented question $x^2 = -1$ had, along with a whole host of others, no answer. For a system supposed to be complete—a system designed to heal—this was a splinter under the nail. An imperfection in a system supposed to be perfect. No number squares to give -1. No number... except (and here we must imagine the imaginary) the mythical quantity $\sqrt{-1}$, whose glyph means simply: "Whatever squares to give -1". So, what's the answer to $x^2 = -1$? Whatever the answer is! *Woop!* Nevertheless, according to the long-established laws of arithmetic, every square is necessarily positive, so, logically, there could be no tally-mark answer to $x^2 = -1$.

There were only two ghost solutions:

$$x = \pm\sqrt{-1} \tag{1}$$

So it was that the spectre of Hippasus returned.

Once again, there was a new number, an alien number, a number than made no sense at all. A beast birthed in old Baghdad. An impossible number! A *devil* number! Only this time, there was no way through. Unlike the earlier demon $\sqrt{2}$, $\sqrt{-1}$ couldn't be squeezed into the gaps. There was no more room at the inn. And, what was worse, nor would there *ever* be. The reals were not just hoped complete, they had been *proved* complete, and this ludicrous fabrication, $\sqrt{-1}$, was demonstrably not one of them.

It is at such moments, when paradox states impossibility, that the greatest opportunity knocks. Only true impossibility allows for true invention. In the West, culturally, we stand at such a juncture; the world, environmentally, stands at such a juncture. And the requirement, when current theories deny all hope of progress, is that culture take a leap of faith, a heresy beyond the last heresy, a transgression against all that is sacred, in order to plant the torch in darkness, in a place that doesn't yet exist. In a place that most, indeed, that almost everyone is convinced *cannot* exist. At the moment when the concepts of the *Weltanschauung* are logically full, the only way out is a leap of faith.

"Je n'ai aucune besoin de cette hypothèse," Laplace said.

And yet, we do have need of it.

The phantom $\sqrt{-1}$ floats in the aether.

G/U is beyond understanding, beyond objective science, beyond... beyond... but *where*? There is no beyond, so the argument goes, as the atheist justifies his rational worldview. Anything "beyond" is necessarily *fairytale*. But the natural, as opposed to supernatural, world is merely "that which has been modelled". What is modelled (thus supposedly controllable) is taken as "natural"; whereas what remains unmodelled (thus supposedly frightening) is "supernatural". Looked at broadly, such blindness can only lead to stagnation. It already has. Our "progress" has been paid for in misery. If we are to heal, we need a worldview of worldviews grander than our current one, and how poignant it is that the complex number, belonging as it does to mathematics, the heir-apparent to reason's crown, should point the way.

Where do we place the beyond?

Elsewhere.

3 The Complex Numbers

In 1545, the Italian polymath Cardano first acknowledged $\sqrt{-1}$, but he only peered through the gate of the labyrinth. He saw glints in the gloom, nothing more. Then, in 1637, the great Descartes described the lurking Minotaur, dubbing it *un nombre imaginaire*. A ghoul to be feared, in other words. Gradually, however, through the course of the 18th century, its use became widespread. It birthed a host of healing theorems. But, all the while, giants such as Euler (who, completing the deeper-Self symbolism, aptly named the unknown i) still saw it as an anathema, and even in 1797, a quarter-millennium after germination, a *full* millennium after the seed was sown, Gauss, perhaps the greatest mathematician of all, was still expressing doubts as to "the true metaphysics of $\sqrt{-1}$."

The fact that this 17th- and 18th-century mathematical struggle regarding the existence of $\sqrt{-1}$ foreshadowed the 20th- and 21st-century psychological battle regarding the existence of the unconscious is no coincidence. Psychology and mathematics are both approaches to the divine: one framed as ego-G/U, the other using the ancient language of number. Thus mathematics reflects psychology, and, in theorem, lemma and corollary, it offers up models for psychic contents. Every piece of mathematics, at root, is a play on the apartness yet togetherness of 1 and 2, the primordial duality expressed in Peano's axioms. The circular structure of waves; the poetry of complex exponentiation; Gödel's Incompleteness Theorems; the unsolvability of the quintic: these are modern *prayers*. The queen of the sciences is, in fact, the Goddess of Metapsychology, and she has a higher purpose than tech.

She is an oracle.

Gauss was to be the prophet.

At last, in 1831, he came to terms with the phantom fully, running with complexity, playing with it, wielding it, and, *post hoc*, as is always the way, reframing the move as inevitable. Once a road is built, it is how it had to be: belief is no longer required to walk it. Describing complex numbers, the great German mathematician wrote: "If this subject has hitherto been considered from the wrong viewpoint and thus enveloped in mystery and surrounded by darkness, it is largely an unsuitable terminology which should be blamed. Had $+1$, -1 and $\sqrt{-1}$, instead of being called positive, negative and imaginary (or worse still, impossible) unity, been given the names, say, of direct, inverse and lateral unity, there would hardly have been any scope for such obscurity."

Lateral unity.

Unity, yes, but *in a different direction.*

Due to the collective work of Descartes, Wallis, Argand and Gauss, $\sqrt{-1}$

was given a place off the number line. (For those unfamiliar with the complex plane: yes, putting a number "off the number line" is every bit as crazy/brilliant as it sounds.) The real numbers run from West to East, but $\sqrt{-1}$ is due North. Beyond, elsewhere, in some ridiculous nowhere. North of the number line was not just unseen ground, but ground that was, depending on the bravery of the critic, either stupid, laughable, impossible, or too insane even to dignify with a response. Only in hindsight, walking in the well-compacted footsteps of faith, is such a move even *thinkable*. The reals had stood intact as the entire numerical universe, since the Stone Age. Talk about "empirically verified". It wasn't even that there was nothing else, there wasn't even language with which to talk about the fact that there was nothing else.

Until, that is, someone placed $\sqrt{-1}$ *laterally*.
Until someone housed it, quite literally, *outside reality.*

4 Conclusion

In the end, all truth expires.

In spite of the denial, in spite of the dogma, the broader answer always grows too tall to hide with "surely not". In mathematics, it has done so a hundred times. In 1797, after some 40,000 years, the linear worldview of "The real numbers are the only numbers" finally gave out, and a broader mathematics appeared, beyond old knowledge, beyond old common sense. But this apparent murder of reason didn't herald destruction. On the contrary, the moment was a beautiful release. There was new soil in which to put down roots. Post-theism, post-atheism: "This World Is All There Is" is creaking, cracking *dogma*. It is yesterday's strength become today's weakness. In a world in which quantum entanglement proves nonlocality, using spatiotemporal perception as justification for "hard rationality" just doesn't make sense any more. It was reason that led us beyond religion, and it is reason that will lead us beyond atheism, to *new* ground.

So, we require a leap of faith.

As primordial unconsciousness once split, day from night, ego from G/U, mathematics bifurcated again. Only this time, not in 1 becoming 2, but in one *dimension* becoming two. The real number line $\mathbb{R}$ became the complex plane $\mathbb{C}$, and a thousand problems sighed to solution. It seems that linearity feels too much like distance. Like juxtaposed Booleanism. Like *agony*. And the division inherent in 1 and 2, when humanly felt, is exactly that. It is the agony of the heart-void, that fate worse than pain. But, the myth tells us, as so many before it, that we cannot retrace our steps. Pills, in the end, solve nothing, and no amount of booze will help; all attempts to mend ourselves from within objective "reality" are doomed. We can only move *laterally*, into uncharted space.

“No such place exists!” the rationalist cries. “Spacetime is complete!” Such is the weeping of Alexander the Great: the desperation of material success and its subsequent misery. But what of the dimension that can’t be seen? What of the unknown? What of the *extended world*? Just because, as real-number consciousnesses, it is impossible to measure complexity, that doesn’t mean complexity doesn’t exist. Existence is a matter of perspective. Quantum mechanics proves that this universe has no objective reality! The subatomic world is pixelated!

In truth, hard reason is no longer a virtue—that of intellectual rigour—it has become *cowardice*. Fear of the unconscious psyche, fear of mystery, fear of what lies behind the cellar door. Over and over and over again, what orthodoxy has proven to be real turns out to be only a subset. Now is no exception. A proof only applies within its own axiomatic system, which means that, in the end, proof proves nothing. $\sqrt{-1}$ was *off the reservation*. The reals were proved complete, yes, but more needed to be done, so we went beyond the reals, to a place called imaginary, complex, *impossible*. And now, as every mathematician knows, the imaginary numbers are every bit as real as the reals were.

What we invent, we become.

Imagine a pendulum, pushed once, swinging gently.

The mathematical approach is: model the scenario, set up some equations, solve them, and predict the behaviour of the idealised system. A simple question: “How far will the pendulum travel before drag brings it to rest?” This is real, applied mathematics. It has an answer that can be tested empirically. But, while restricted to the reals, the real differential equation that emerges from Newton’s Second Law is unsolvable: in underdamped harmonic motion, the single dimension of the old number line is insufficient for purpose. Real mathematics is like a journey along a country road in a low-slung sports car: perfect as long as there are no obstacles, but, where a fallen tree blocks the way, useless. The pendulum problem is a real question, but it isn’t solved by clinging to ”reality”.

It *returns* to reality.

There’s the leap of faith.

As soon as we extend the known, broadening the scope of enquiry from the real road to the complex wilds surrounding it (nonexistent ground, according to the sports car) we are presented, almost immediately, with elegant solutions. Mathematical *beauty*. The 4x4 waves goodbye to the sports car, and dives through the hedge. In the pendulum problem, the characteristic polynomial, which had no solutions in real numbers, now has two in $\mathbb{C}$, and Euler’s method for complex exponentiation (which is rightly recognised as poetry) yields a pair of 4x4 solutions: real, onroad question; complex, offroad method.

But what use is an answer in higher dimensions? Of what benefit is G/U for the business of *reality*? Surely, postulating further dimensions solves no problems. Real pendulum, real road, real job, real pain: how does extension help with the mortgage, with a leaking roof, with divorce, with despair? What good is fiction in response to fact? Well, the answer must be no good at all, if we remain floating in some philosophical hinterland, waxing lyrical about imaginary phantoms. A jeep that leaves the road for good is lost. This is going native, staying complex, failing to return from newfound depth, and it is even more foolhardy than giving up and turning round.

In applied mathematics, such complex solutions are always recombined, are always *re-realised*, so as to produce earthy results. Answers for the real world. The jeep spends as little time offroad as possible. It circumvents the tree, using the deeper dimension, then returns to the tarmac. Those who open their minds, but forget to return, are fools: the real world is where life happens. $\mathbb{C}$ offers depth, then depth's *return.* Thus the sage becomes enlightened, but doesn't speak in tongues. The hero ascends the mountain, but then returns to the valley. The lover of life doesn't disappear into solitary, buy a kaftan and start wittering about enlightenment, the lover of life *lives.* The mathematician goes into the complex plane, and then *returns.*

Real questions; real answers.

Just as alchemy morphed into chemistry, pure mathematics has morphed into science, but its purpose was never such mastery, not of the outer world, at least. The higher-level story of the queen of the sciences has more to teach us than this or that technique. In the end, who cares about the *facts* of mathematics? It is the *journey* that is everything. The queen, behind her veil of rationality, is none other than the goddess of fate, and the joy she brings—the *awe*, indeed—is in the fact that she requests and requires *faith*: in the motion of pendula, the Fundamental Theorem of Algebra, the complex isomorphisms of aeronautics. Mathematics is, in fact, a soothsayer, a tool for the raising of consciousness, a grand myth of reality, and more. Question and answer lie in $\mathbb{R}$. But the path between them?

It lies *beyond.*